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ABSTRACT

These instructional objectives have been selected from materials submitted to the Curriculum Laboratory of the Graduate School of Education at UCLA. Arranged by major course goals, these objectives are offered simply as samples that may be used where they correspond to the skills, abilities, and attitudes instructors want their students to acquire. These objectives may also serve as models for assisting instructors to translate other instructional units into specific measurable terms. For other objectives in related courses see: ED 033 683 (College Algebra); ED 033 687 (Calculus and Analytic Geometry); ED 033 698 (Geometry); JC 710 129 (Intermediate Algebra); and JC 710 130 (Introduction to Mathematical Thinking). (MB)

Instructional Objectives for a Junior College Course in College Mathematics

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ERIC Clearinghouse for Junior Colleges
University of California
Los Angeles, California

June 1971

UNIT I: THE NATURE OF MATHEMATICS

Major Concepts:

Mathematics as an abstract science in its own right

Mathematics as a tool for the scientist: Mathematical models

Mathematical logic and language: Negations; Implications

Methods of proof and disproof

Unit Objectives:

Goal: The student will understand the necessity for an engineer to be able to understand mathematics rather than just substitute into equations.

Goal: The student will realize that mathematics is independent of the physical world and that mathematical statements are either true or false.

1. Outside of class, the student will write an essay of 200-500 words on the following questions: "Why is mathematics considered completely abstract, whereas other sciences are not? What is the difference between mathematical truth and physical truth? How could such an abstract system be of any use at all to an engineer? Why should an engineer bother learning mathematical techniques (for example, derivation of formulas) rather than simply the formulas he will need in his own work?" The answer will include statements concerning: mathematical dependence only on axioms, science's dependence on the physical world, mathematical truth by proof from axioms, scientific truth by observation, mathematical models, insufficiency of present formulas for future situations.

80%

Goal: The student will negate given statements.

2. Given a closed sentence, the student will give its negation. (Examples are on page 8 of the text).

75%

Goal: The student will make open sentences into true closed sentences by using such phrases as:"for all x", "for some x", "for no x".

3. Given an open sentence, the student will give a corresponding true closed sentence by using the phrases "for all x", "for some x", "for no x".

Example: Open: If x is even, then x+1 is odd.

Closed: For all x, if x is even, then x+1 is odd.

80%

Goal: The student will form converses and contrapositives of given implications.

4. Given an implication, the student will give its converse. Given an implication, the student will give its contrapositive.

Examples: A polygon is a triangle if it has three sides.

If x is even, then x+1 is odd.

For a number to be even, it is sufficient for its square to be even.

75%

Goal: The student will realize that even though an implication is true, its converse may not be , but the contrapositive is true.

5. The student will state whether the following statements are true or false, and if false (the first two), he will give a counterexample.

If an implication is true, then its converse is true.

If an implication is false, then its converse is false.

If an implication is true, then its contrapositive is true.

If an implication is false, then its contrapositive is false.

75%

Goal: The student will put statements into implication form.

6. Given an implication, the student will put it into "if... then" form.

80%

Goal: The student will recognize and construct valid direct proofs, indirect proofs and disproofs.

7. Given proofs, valid or invalid, the student will tell whether they are valid or invalid, and if valid, he will tell what type (direct or indirect) they are.

85%

8. Given true statements, the student will construct a valid proof (either direct or indirect).

85%

9. Given a false statement, the student will construct a valid disproof.

90%

UNIT II: THE REAL NUMBER SYSTEM

Major Concepts:

Properties of the real number system

Mathematical induction

Absolute value

Unit Objectives

Goal: The student will know the definitions and uses of natural numbers, integers, rational numbers, and real numbers.

10. Given one of the following terms--natural number, integer, rational number, real number--the student will give the definition.

90%

11. The student will give examples of problems that can be solved in (1.) the integers, but not the natural numbers, (2.) the rationals, but not the integers, (3.) the real numbers, but not the rationals.

75%

Goal: The student will recognize the properties of the real numbers and use them in proofs and in problems.

12. The student will list and define the eleven basic properties (field axioms) for the real numbers.

80%

13. Given a set with a binary operation on it and one of the basic properties, the student will state whether or not the operation has that property. If not, he will give

an example where it fails.

Examples: Is subtraction of real numbers commutative?

Possible answer: No, because $2-0=+2$ and $0-2=-2$.

Let us define * on the integers as: $a*b=a+2b$. Is * associative? Possible answer: No, because $1*(1*1)= 1*(1+2)=1+2\cdot3=7$, yet $(1*1)*1=3*1=3+2\cdot1=5$.

80%

14. Given a true statement involving real numbers, the student will prove it using only the basic properties (field axioms).

Examples: If a, b, c, are real numbers and $a+b=c+b$, then $a=c$.

If a and b are positive numbers, then $a\times(-b)$ $-(ab)$.

75%

15. Given a number, the student will give its additive inverse; given a number $\neq 0$, the student will give its multiplicative inverse.

95%

16. The student will evaluate expressions such as $-7(2(3+4)-1)+4(3-6(2+1))$.

90%

<u>Goal</u>: The student will understand absolute value.

17. The student will define the absolute value.

100%

18. The student will find the absolute value of a given number.

95%

19. The student will give the rules for addition of signed numbers using the absolute value (theorem 2, page 24 in the text).

100%

20. Given a statement involving absolute value, the student will prove it or disprove it.

Examples: $|a| + |b| \geqslant |a+b|$.

$|a-b| = |a| - |b|$.

$|a|^2 = |a^2|$.

$|a| + |a| = |a+a|$.

75%

Goal: The student will understand mathematical induction.

21. Given a statement involving integers (such as those in problems 2.5), the student will prove it if it is true and show where mathematical induction fails if it is false.

80%

22. At home, the student will write a discussion of what the fault is in the following "proof" by mathematical induction: Theorem: All horses are the same color. First we prove that if there are n horses, they are all the same color. n=1: If we have one horse, it is certainly the same color as itself. Assume the statement for k: If you have k horses, they are all the same color. Now to prove it for k+1. Suppose you have k+1 horses. Take one away. Then you have k horses, so they are all the same color. Take another one away and bring back the first one. Again you have k horses, so they are all the same color. Therefore, all k+1 horses are the same color. Therefore, the statement is true for all n.

Therefore, all horses are the same color.

50%

<u>Goal:</u> The student will understand the special properties of zero.

23. The student will define each of the following forms or explain why it cannot be defined: $\frac{0}{b}$, $\frac{a}{0}$, $\frac{0}{0}$, where $a \neq 0$, $b \neq 0$.

90%

24. The student will prove: If a and b are two real numbers such that $a \cdot b = 0$, then $a = 0$ or $b = 0$.

70%

<u>Goal:</u> The student will prove that 2 is irrational.

25. The student will prove that if a^2 is divisible by 2, then a is divisible by 2.

80%

26. The student will prove that 2 is not rational.

75%

UNIT III: PCLYNOMIALS

Major Concepts:

Operations with polynomials

Binomial theorem

Factoring

Unit Objectives:

Goal: The student will recognize polynomials, add them, multiply them, and be familiar with the terminology related to them.

27. Given an algebraic expression, the student will tell whether or not it is a polynomial.

95%

28. Given two polynomials, the student will give their sum.

95%

29. Given two polynomials, the student will give their product.

90%

30. The student will define polynomial, degree, term, coefficient.

100%

Goal: The student will solve problems using the binomial theorem.

31. The student will compute $(a+b)^n$ where a and b are monomials and n is a positive interger. Example: $(3x^2-y)^6$

75%

32. The student will state the binomial theorem, defining the expressions $\binom{n}{r}$ and n!

85%

33. The student will compute a particular term of $(a+b)^n$ where a and b are monomials and n is a positive integer.

75%

34. The student will compute a given power of a given trinomial. Example: $(x+y+z)^3$

50%

<u>Goal</u>: The student will divide polynomials.

35. Given two polynomials $D(x)$ and $P(x)$, the student will find two more polynomials $Q(x)$ and $R(x)$, such that $P(x) = D(x) \cdot Q(x) + R(x)$ and the degree of $R(x)$ is less than the degree of $D(x)$. 85%

<u>Goal</u>: The student will factor polynomials.

36. Given a trinomial of the form x^2+bx+c, b,c integers, the student will factor it into two binomials (with integer coefficients) if this is possible, and if it is impossible will show that it cannot be done.

90%

37. The student will do the same as above, given a trinomial of the form ax^2+bx+c, where a,b,c, are intergers.

80%

38. The student will factor a given polynomial of the special forms: a^2-b^2, a^3+b^3, and a^3-b^3 where a and b are algebraic expressions. 80%

UNIT IV: ALGEBRAIC FRACTIONS

Major Concepts:

Simplifying algebraic fractions

Operations with algebraic fractions

Unit Objectives:

Goal: The student will understand simplification of algebraic fractions.

39. Given an algebraic fraction, which may or may not be in lowest terms, the student will reduce it to lowest terms. 85%

40. The student will give a counterexample to the statement: "If k and a are algebraic expressions, then $\frac{k+a}{k+b} = \frac{a}{b}$." 100%

41. The student will give a counterexample to the statement: "If k and a are algebraic expressions, then $\frac{k+a}{k} = a$." 100%

Goal: The student will add, multiply and divide polynomial expressions.

42. Given two or more polynomial fractions, the student will give their least common denominator. 80%

43. Given two or more polynomial fractions, the student will give their sum in simplest form. 80%

44. Given two polynomial fractions, the student will give their product in simplest form. 80%

45. Given two polynomial fractions, the student will give

their quotient in simplest form.

80%

46. Given an expression involving polynomial fractions and more than one of the above operations, the student will put the expression in simplest form.

Example: $\left(\frac{x+1}{x-1} + \frac{2x+3}{x^2-1}\right) x \left(\frac{1}{x} - \frac{2x+1}{x^2}\right)$,

$$\frac{\frac{1}{x+1} - \frac{1}{x-1}}{\frac{1}{x+1} + \frac{1}{x-1}}$$

75%

UNIT V: EXPONENTS AND RADICALS

Major Concepts:

Law of exponents

Negative and fractional exponents

Even and odd roots

Unit Objectives:

Goal: The student will understand the basic laws of exponents: $a^m \times a^n = a^{m+n}$, $(a^m)^n = a^{mn}$, $(ab)^n = a^n \times b^n$

47. Given a problem of the form: $a^n \times a^m = a^?$ or $(a^m)^n = a^?$ the student will give the exponent that has been left out. Example: $2^{\frac{1}{2}} \times 2^{\frac{1}{4}} = 2^?$, $(10^8)^2 = 10^?$, $x^r \times x^s = x^?$, $5^{-1} \times 5^1 = 5^?$

95%

48. The student will give a counterexample to the false statement: $(a^m)^n = a^{(m^n)}$

100%

49. The student will choose the correct answer to the following question: "What does the basic law for multiplying powers of numbers (theorem 1, Chapter 5) tell us about $3^2 \times 2^3$?"

* a. nothing

b. $3^2 \times 2^3 = 6^5$

c. $3^2 \times 2^3 = 6^6$

d. $3^2 \times 2^3 = 3^5$

e. $3^2 \times 2^3 = 2^5$

100%

Given what type (positive integers, nonzero integers, rationals) the student will tell for which "a" the law of exponents holds.

100%

51. Given a power of a product such as $(2x)^3$, the student will give the corresponding product of powers $(2^3 \times x^3)$.

90%

52. Given a product of two terms to the same exponent, the student will give the corresponding power of a product.

90%

53. The student will simplify expressions involving exponents. Example: $2^4 \times 5^{-2} \times 5^3 \times 2^{\frac{1}{2}}$.

90%

Goal: The student will convert expressions involving fractions (radicals) into expressions involving negative (fractional) exponents, and vice versa.

54. Given an expression involving negative exponents, the student will give the corresponding expression involving only positive exponents.

Example: $\dfrac{x^{-1}y + 3x - y^{-1} + 2^{-2}}{x^{-1}y^{-3}}$

85%

55. Given an expression involving fractions, the student will give the corresponding expression involving negative exponents.

Example: $\frac{x}{y}\left(\frac{3}{x} + \frac{1}{y} + 2\right)$

85%

56. Given an expression involving radicals, the student will give the corresponding expression involving fractional exponents.

80%

57. Given an expression involving fractional exponents, the student will give one or more corresponding expressions involving radicals. Example: $2^{3/4} = \sqrt[4]{2^3} = (\sqrt[4]{2})^3$

85%

<u>Goal</u>: The student will be familiar with the problems involved in taking roots.

58. The student will discuss the statement: $(a^2)^{\frac{1}{2}} = a$. His answer will include the following: It is true only for $a \geq 0$, and that for $a < 0$, $(a^2)^{\frac{1}{2}} = -a$. In general, $(a^2)^{\frac{1}{2}} = |a|$.

80%

59. The student will define $(-a)^{\frac{1}{2}}$ where $a>0$.

75%

60. The student will give a counterexample to the following statement: "For every a and b nonzero integers, $a^{\frac{1}{2}}b^{\frac{1}{2}} = (ab)^{\frac{1}{2}}$."

75%

61. The student will define $(-a)^{p/q}$ where p is an odd integer, q is an integer and $a>0$.

75%

62. The student will select from a list those expressions which can be defined as real numbers. Examples: $(-1)^{\frac{1}{2}}$, $(-17)^{1/3}$, $(3)^{2/3}$, $(-2)^{\frac{1}{4}}$.

80%

<u>Goal</u>: The student will simplify expressions involving positive and negative rational exponents and/or radicals.

63. The student will simplify expressions containing positive and negative exponents, such as, $\dfrac{x^2 + 2x^{-1} - x^{-\frac{1}{2}}}{x^{\frac{1}{4}}}$ 80%

64. The student will simplify expressions involving the exponent $\frac{1}{2}$, where the numbers involved may be negative or zero. (The answer will generally involve the absolute value). Example: $(x^2+2x+1)^{\frac{1}{2}} + (x^2-2x+1)^{\frac{1}{2}} = |x+1| + |x-1|$

75%

65. Given an expression with a radical in the denominator, the student will give an equivalent expression with no radical in the denominator. Examples: $\frac{1}{\sqrt{2}}$, $\frac{3}{\sqrt{2}+\sqrt{3+x}}$

80%

66. Given two or more expressions involving radicals in the denominator, and a given operation (addition, subtraction, multiplication or division), the student will perform the operation. Example: $\frac{1}{1+\sqrt{x}} \div \frac{\sqrt{x+1}}{\sqrt{4x}}$

80%

UNIT VI: SETS AND EQUATIONS

Major Concepts:

Set theory--set, subset, intersection and union.

Equivalent equations.

Quadratic formula.

Solution of equations containing fractions and radicals.

Unit Objectives:

Goal: The student will be familiar with the notation and terminology of set theory.

67. The student will define each of the following, where A and B are sets: $\emptyset$, $A = B$, $A \subseteq B$, $A \subset B$, one-to one correspondence between A and B, $A \cap B$, $A \cup B$.

90%

68. The student will give examples of sets A and B, $A \subset B$ such that there is a one-to one correspondence between them.

100%

69. Given a list of sets, the student will identify the identical pairs.

95%

70. Given a pair of sets, the student will give a one-to-one correspondence between them, if possible.

75%

71. Given a pair of sets, the student will give their intersection.

95%

72. Given a pair of sets, the student will give their union.

95%

<u>Goal</u>: The student will understand the meaning of equivalent equations.

73. Given an equation, the student will find its solution set.

90%

74. Given two equations, the student will tell whether or not they are equivalent.

90%

75. The student will give an example of polynomials $P(x)$, $F(x)$, and $G(x)$ such that $P(x) \cdot F(x) = P(x) \cdot G(x)$ has a different solution set from $F(x) = G(x)$.

100%

76. The student will give an example of polynomials $F(x)$ and $G(x)$ such that $\{x \mid F(x) = G(x)\} \neq \{x \mid (F(x))^2 = (G(x))^2\}$.

100%

<u>Goal</u>: The student will prove and use the quadratic formula.

77. The student will write a proof of the quadratic formula.

90%

78. Given a quadratic equation, the student will give both roots.

90%

79. Given a quadratic, the student will factor it into two first degree polynomials.

85%

80. Given a quadratic equation, the student will tell whether the roots are real or imaginary and whether they are equal or not, without actually solving for the roots.

95%

81. Given an equation containing fractions such as $\frac{x}{x+2} - \frac{4}{x+1} = \frac{-2}{x+2}$, the student will solve for all possible solutions and from these choose the true solutions, if any. (Equating the numerators over the GCD may give extraneous roots.)

80%

82. Given an equation containing one or two radicals, the student will solve for all possible solutions and from these choose the true solutions, if any. (Squaring may give extraneous roots.)

80%

UNIT VII: SIMULTANEOUS EQUATIONS AND MATRICES

Major Concepts:

Solutions of simultaneous linear equations--algebraically, graphically, and by Cramer's Rule.

Vectors

Matrices

Determinants

Unit Objectives:

Goal: The student will solve n simultaneous linear equations in n unknowns.

83. Given two simultaneous linear equations in two unknowns, the student will give a series of equivalent pairs of equations that lead to the correct solution, if one exists (i.e., solve them algebraically).

85%

84. Given two simultaneous linear equations in two unknowns, the student will graph both equations and give their solution as a point on both graphs.

80%

85. Given three simultaneous linear equations in three unknowns, the student will solve them algebraically, provided a unique solution exists.

80%

Goal: The student will understand the basic ideas of vector notation and usage.

86. The student will define vector, x-component, y-component, equality of vectors.

100%

87. Given two vectors in component notation, the student will give their sum.

95%

88. Given two vectors represented as directed line segments in the plane, the student will give their sum as a directed line segment in the plane.

90%

89. Given a vector in component notation, the student will give the corresponding vector in $\vec{i}$, $\vec{j}$, $\vec{k}$ notation, and vice-versa.

95%

90. Given two vectors in component notation, the student will give their inner product.

90%

91. Given a vector, the student will give its length.

90%

92. Given two three-dimensional vectors, the student will give their outer product.

80%

93. Given a field axiom, the student will prove or disprove it for vectors under addition and inner or outer multiplication.

70%

Goal: The student will understand the basic ideas of matrix notation and usage.

94. The student will define matrix, equality of matrices, singular and nonsingular.

100%

95. Given two n×m matrices, the student will give their sum.

90%

96. Given an n×m matrix and an m×1 matrix, the student will give their product.

85%

97. Given a field property, the student will prove or disprove it for matrices. For example, the student will give counterexamples to statements such as: "Matrix multiplication is commutative." "If A and B are matrices such that AB = 0, then A = 0 or B = 0."

70%

<u>Goal</u>: The student will understand determinants and recognize the difference between matrices and determinants.

98. Given a 2×2 or 3×3 matrix, the student will give its determinant.

90%

99. The student will choose the correct answer on questions that ask him to distinguish between matrices and determinants, such as:

"Let $A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$. Then a.) matrix A = matrix B. *b.) det A = det B. c.) Both of these. d.) Neither of these."

"Choose the correct statement: a.) Multiplication of matrices is commutative. *b.) Multiplication of determinants is commutative. c.) Neither of these. d.) Both of these.

100%

100. The student will prove or disprove statements about determinants such as: $\det\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} = \det\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$, in which the determinant of a matrix is claimed to be unchanged by some rearrangement of the elements, adding certain elements together or multiplying certain elements by a constant.

70%

101. Given two 2×2 or 3×3 matrices, the student will verify that det(A) · det(B) = det(AB).

85%

<u>Goal</u>: The student will invert nonsingular matrices.

102. Given a matrix (2×2 or 3×3) the student will give its determinant and if it is not zero, he will solve for the inverse of the matrix.

80%

<u>Goal</u>: The student will solve n simultaneous linear equations in n unknowns using Cramer's Rule.

103. Given n linear equations in n unknowns (n = 2 or 3), the student will give the corresponding matrix equation AX = -D and solve for the vector X.

85%

104. The student will give the family of vectors whose inner product with each of two given vectors is zero.

80%

<u>Goal</u>: The student will solve "word problems" that can be translated into simultaneous linear equations, especially those from science and engineering.

105. Given a word problem such as those on pages 149-51 in the text (especially 6, 9, 10, 11, 13, 14, and 15.), the student will give the corresponding simultaneous equations and then solve them.

85%

UNIT VIII: INEQUALITIES

Major Concepts:

Laws governing inequalities

Algebraic solution of inequalities

Solving inequalities by graphing

Unit Objectives:

Goal: The student will work with inequalities.

106. Given a list of inequalities, the student will identify the equivalent pairs.

90%

107. The student will prove or disprove statements about inequalities such as: "If $a<b$ and $c<d$, then $a+c<b+d$." If $a<b$ and c is any nonzero number, then $ac<bc$."

80%

Goal: The student will solve inequalities in one variable.

108. Given a linear inequality in one variable, the student will give a series of equivalent inequalities which lead to the correct solution set.

90%

109. Given a quadratic ax^2+bx+c, the student will solve for the solution set of $ax^2+bx+c \geq 0$ (or ≤ 0, or >0, or <0).

85%

110. The student will answer a question such as the following: "The solution set to $(x-1)(x+2)>0$ is $\{x|x>1\} \cup \{x|x<-2\}$. Why are there two pieces? What does each piece represent and how is that related to our problem?" A suitable answer would include the facts that if $(x-1)(x+2)>0$, then $x-1>0$ and $x+2>0$ or $x-1<0$ and $x+2<0$, and $\{x|x>1\} = \{x|x-1>0 \text{ and } x+2>0\}$; $\{x|x<-2\} = \{x|x-1<0 \text{ and } x+2<0\}$

100%

111. Given an inequality in one variable, the student will graph the solution set on the line.

90%

112. The student will prove theorems involving inequalities, such as: $\sqrt{ab} \leq \frac{a+b}{2}$, where $a > 0$ and $b > 0$; $(a_1b_1+a_2b_2)^2 \leq (a_1^2+a_2^2)(b_1^2+b_2^2)$.

40%

Goal: The student will solve inequalities in two variables by graphing.

113. The student will graph a linear inequality in the plane, explaining why it is on the side of the line that he has indicated.

80%

114. The student will graph two or more simultaneous inequalities of the form $ax+by+c \geq 0$ (or ≤ 0, or > 0, or < 0.) in the plane, including graphs of the boundary lines.

80%

Goal: The student will solve inequalities of the type $ax+b \geq c$ (or $\leq c$, or $> c$, or $< c$).

115. Given an inequality of the type above, the student will give the corresponding linear inequalities for the cases $ax+b \geq 0$ and $ax+b < 0$.

90%

116. Given an inequality of the type above, the student will give the solution set.

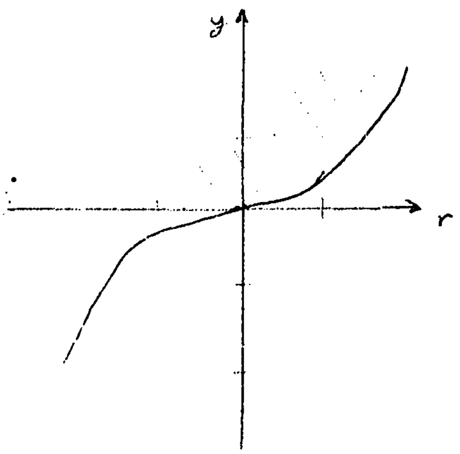
85%

117. The student will solve practical problems involving inequalities, such as:

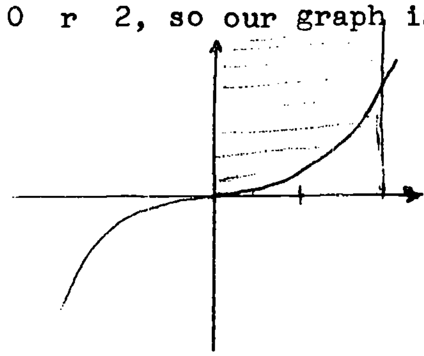
"A student who does not know that the volume of a sphere is $4/3\pi r^3$ is measuring a heavy sphere by water displacement. He has a graduated cylinder 4 inches in diameter, filled with water to within v inches of the top. The volume of the sphere is determined by the change in the water level when the sphere is put in the cylinder. Unfortunately, if the sphere is too large, the water will overflow and no measurement can be made. Also, it must be small enough so that it fits into the cylinder. What are the restrictions on the radius of the sphere and the height of the water below the top of the cylinder? Make a graph.

Solution: The radius of the cylinder $= 2 > r$, since otherwise the sphere would not fit. But the radius must be positive. Therefore, $0 < r < 2$. This is one inequality that must be satisfied. The empty volume left in the cylinder is $v\pi 2^2$ (the volume of a cylinder of height v and radius 2). Therefore, the volume of the sphere must be $< 4\pi v$. But the volume is $4/3\pi r^3$. So $4/3\pi r3 < 4\, v$. So we get another inequality, $\frac{r^3}{3} < v$.

Here is its graph:



But we also must have 0 r 2, so our graph is:



75%

UNIT IX: FUNCTIONS AND RELATIONS

Major concepts:

Set theoretic definitions of relation and function.

Algebra of functions

Graphs

Unit Objectives:

Goal: The student will be familiar with the vocabulary and set theoretic notation for relations and functions, as well as the difference between a relation and a function.

118. The student will define relation, function, domain, range and composite of two functions.

100%

119. The student will give an example of a finite relation.

100%

120. The student will give an example of a relation defined by an equation or inequality.

100%

121. Given the defining rule of a relation or an entire finite relation, the student will give the domain and range.

90%

122. The student will give an example of a relation that is not a function.

100%

123. Given a finite relation written in table form, the student will give the corresponding ordered pairs.

90%

124. Given a list of relations, the student will identify those that are functions

90%

125. Given a word problem such as problems 29-31 on pages 171-2, the student will give the corresponding function, its domain and range. 80%

<u>Goal</u>: The student will add, subtract, multiply and divide functions, as well as evaluate and compose them.

126. Given two functions, the student will give their sum, difference, product or quotient and the domain and range of the new function.

85%

127. Given two functions, the student will give their composition and the domain and range of the composite function.

85%

128. Given a function and a number in the domain, the student will give the value of the function there. Example, Evaluate $|x^2-4x|$ at $x = 1, -3, 0$.

90%

129. The student will prove or disprove statements about functions such as: If f and g are two functions, then f+g=g+f; if f and g are two functions, the $f \circ g = g \circ f$; If f has domain d_f and g has domain d_g, then $f \circ g$ has domain $d_f \cdot d_g$.

80%

<u>Goal</u>: The student will define and use the idea of a linear relation.

130. The student will define the term "linear relation" and give an example.

131. Given a description of a situation in which two things are related linearly and some values of this relation, the student will give the linear relation in functional notation. Example: Problem 6 below.

80%

<u>Goal</u>: The student will be familiar with various ways of visualizing functions and relations.

132. Outside of class, the student will write an essay of 100-300 words in which he discusses various ways of visualizing a relation and the special case of a function.

75%

<u>Goal</u>: The student will graph functions and relations.

133. Given a function, the student will solve for its intercepts.

90%

134. Given a function, the student will solve for its horizontal and vertical asymptotes.

90%

135. Given a function, the student will tell whether or not it is symmetric with respect to the x-axis, the y-axis or the origin.

85%

136. Given a function, the student will construct a table of values (at least five values if the function is not linear, at least three if it is) and plot the points on a graph.

80%

137. Given a function, the student will sketch a graph which includes indications of all horizontal and vertical

asymptotes and at least five plotted points.

70%

138. Given a relation, the student will sketch its graph. Examples: linear and quadratic inequalities.

80%

<u>Goal</u>: The student will understand inverse functions.

139. Outside of class, the student will write an essay of 150-400 words in which he discusses the various ways of visualizing the inverse of a function. This will include ordered pairs "turned around", mapping going in the other direction, machine running backwards and a graph rotated 180° about the line $y=x$. He will use each of these to demonstrate that $(f^{-1})^{-1} = f$.

75%

140. The student will give an example of a function with no inverse function.

100%

141. Given a function of the form $y=f(x)$ and its domain and range, the student will solve for f^{-1}, if it exists, and give its domain and range.

95%

142. Given a function, the student will define the domain so that the function has an inverse and solve for this inverse. Examples: Illustration 2, page 187 of the text, problem 13 below.

80%

<u>Goal</u>: The student will derive functions from equations.

143. Given an equation such as $2x^2+y^2=1$, $x + y =1$ or $x^2-2y = 0$, the student will give one function (or more than one if necessary) of x in terms of y and another (or more than one if necessary) of y in terms of x, along with suitable ranges and domains.